Developing Understanding of Number and Calculations

Gorse Hall Primary School Calculation Policy

This policy supports the White Rose maths scheme used throughout the school. Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum. This policy has been designed to teach children using concrete, pictorial and abstract representations. This allows the children to show their thinking, check their calculations and develop a deeper understanding of number and calculation.

Mathematics mastery

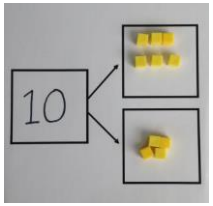
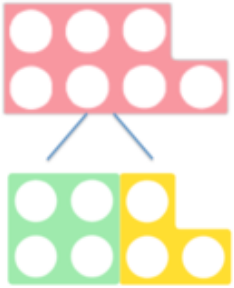
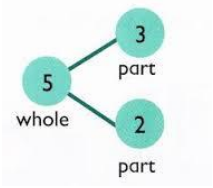
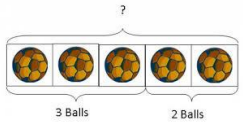
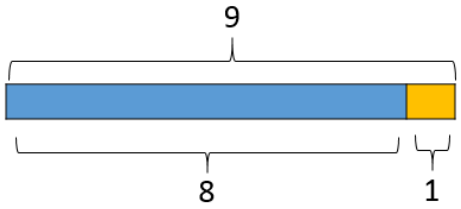
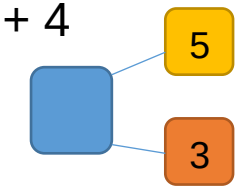
The mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should all have access to the same curriculum content, and rather being extended with new learning, they should deepen their conceptual understanding by answering varied problems and reasoning questions. The children will demonstrate their understanding of the different calculation strategies using concrete and pictorial representations. Reinforcement is achieved by going back and forth between these representations. High expectations of the mathematical vocabulary used are essential, with teachers only accepting what is correct.

How to use the policy:

This policy has been set out as a progression of mathematical skills and not into year group objectives to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or when to move on to the next concept. Children should not be extended with new learning before they are ready, they should deepen their contextual understanding with reasoning questions and challenging problems. All teachers have been given the planning from the White Rose Maths Hub and are required to base their planning around their year group's modules and not to move on to a higher year group's scheme work.

Teachers can use any teaching resources that they wish to use and the policy does not recommend one resources over another, rather that, a variety of resources are used. For each of the four rules of number, a variety of strategies are laid out including concrete materials which can be used along with suggested pictorial representations. This then leads to an abstract representation where a child is capable of representing problems using mathematical notation.

Addition

Objective , Strategies and vocabulary	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model part, whole, add on, more than, increase, bigger, altogether	   Use cubes to add two numbers together as a group or in a bar.  Part - Whole 	   Use pictures to add two numbers together as a group or in a bar.  A bar model which encourages the children to count on in ones.	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.

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Starting at the bigger number and counting on

counting on, number line, bar model, bead strings, numicon, abacus.

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Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.

Use cubes, numicon, abacus or a bead string.

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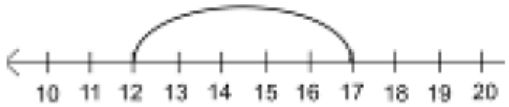
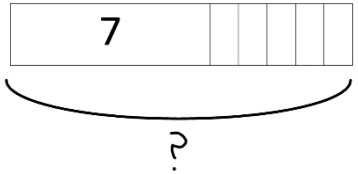
12 + 5 = 17



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

31 + 4 = 35

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



A bar model which encourages the children to count on.

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5 + 12 = 17

Place the larger number in your head and count on the smaller number to find your answer.

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Regrouping to make 10.

number bonds, abacus, numicon, ten frame

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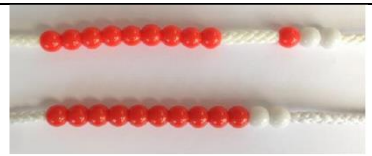

6 + 5 = 11

Start with the bigger number and use the smaller number to make 10.

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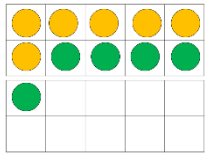
Use pictures or a number line. Regroup or partition the smaller number to make 10.



Children to draw the ten frame and counters.

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7 + 4= 11

If I am at seven, how many more do I need to make 10? How many more do I add on now? 7 +____ = 10

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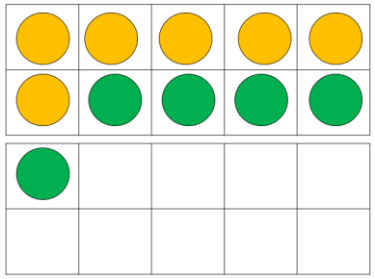
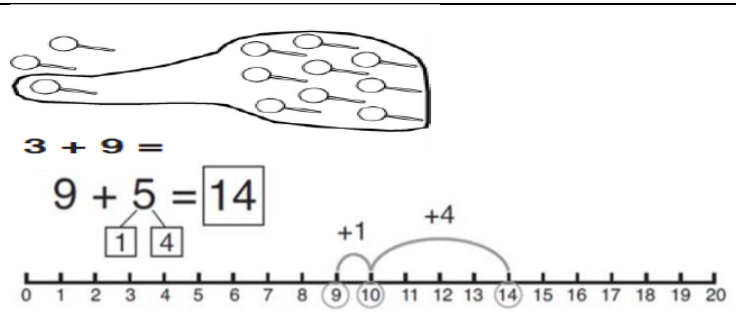
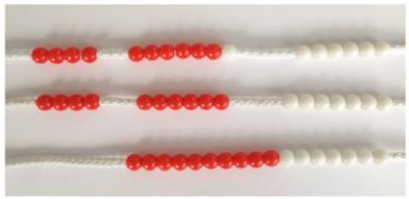
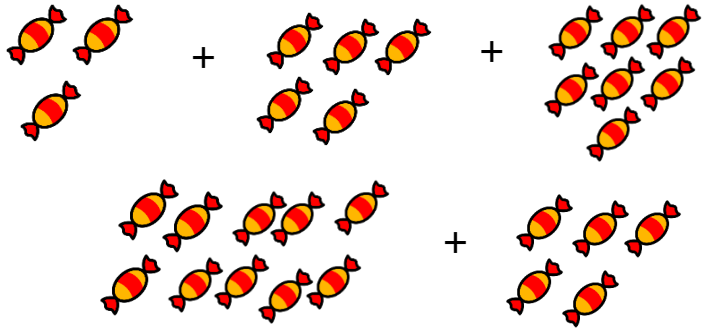
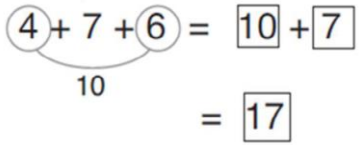
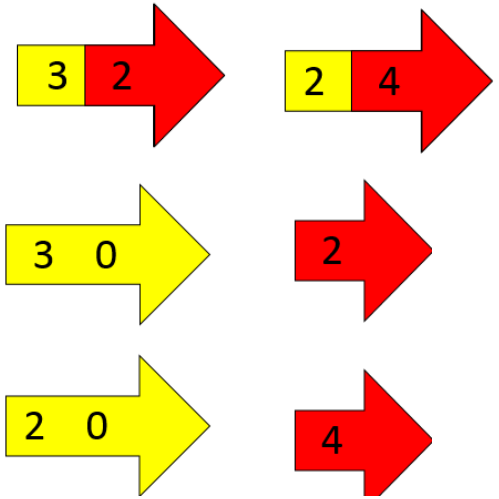
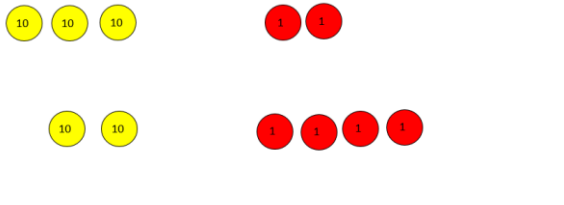
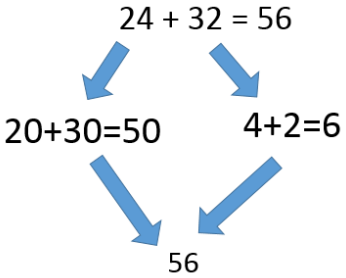
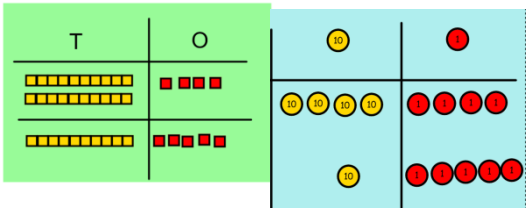
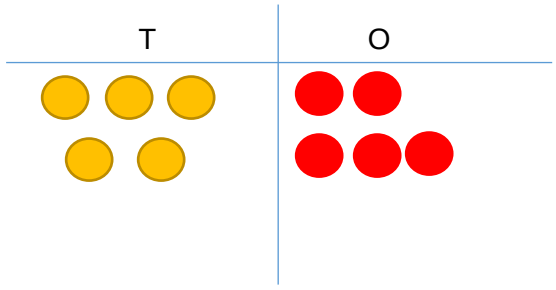
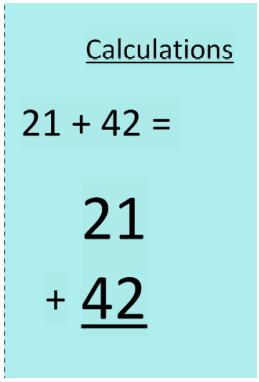
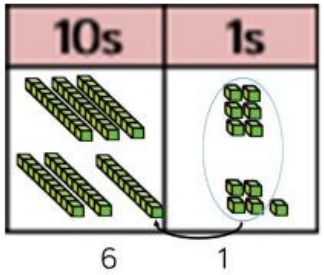
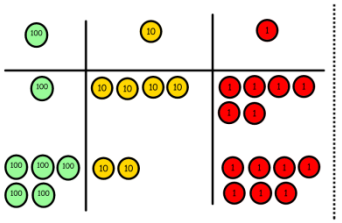
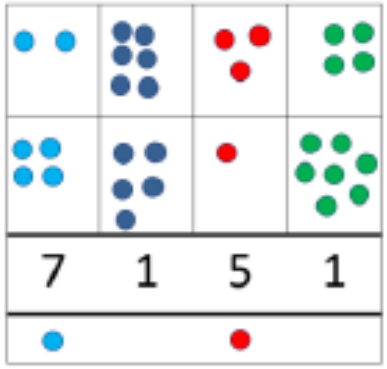
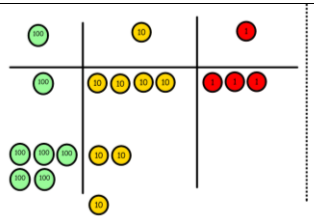
			
Adding three single digits Increase, more than, number bonds, altogether,	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Use numicon to visualise:  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	Draw ten frame, ones and tens rods, and counters.  Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

	Also, use ten frame and abacus to visualise calculation.		
Mentally adding two-digit numbers by partitioning. bridging, partitioning, ones, tens, altogether, place value	Partitioning (and bridging ten) 32+24 = 56 	Drawing counters or base ten. 	Partitioning, adding, then recombining. Adding the ones together first and then adding the tens together. 
Column method- no regrouping addition, plus, altogether, increase	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters and the place value chart to help them to solve additions. 	

Column method- regrouping addition, plus, altogether, increase, exchange	Children should be familiar with the use of 1s/Ones/O. TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25  Ten 1s exchange for one 10. Make both numbers on a place value grid.  146 + 527 Add up the ones and exchange 10 ones for one 10.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. Children draw on the exchange at the bottom of the place value chart.  	Clearly show the exchange below the addition. 536 + 85 621 1 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2



146
+ 527

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

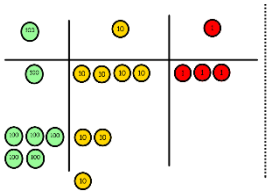
This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

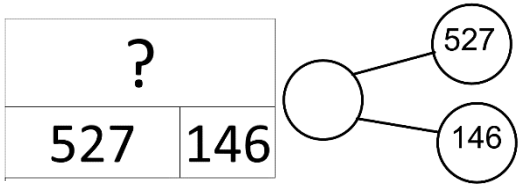
Conceptual variation

Use of bar models, part whole models, place value charts, word problems and inverse calculations.

146+ 527=



146
+ 527



One hundred and forty six + five hundred and twenty seven =

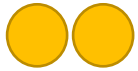
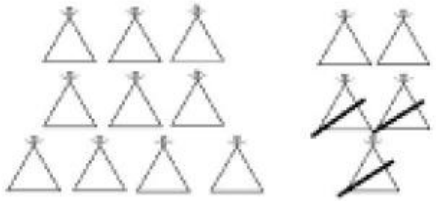
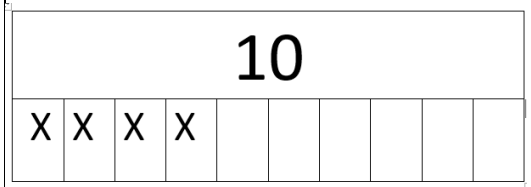
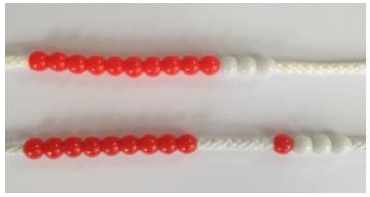
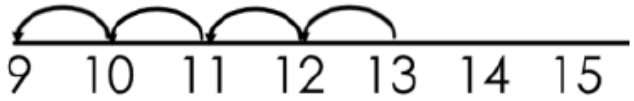
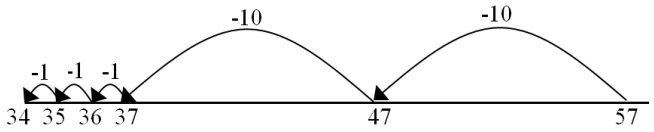
A zoo has 526 animals. In the following year, another 146 arrive. How many are there altogether?

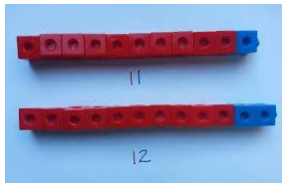
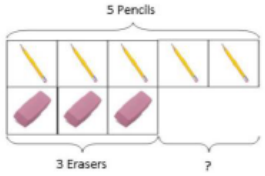
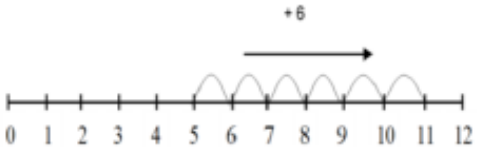
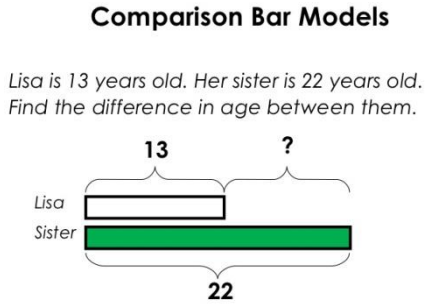
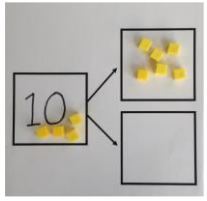
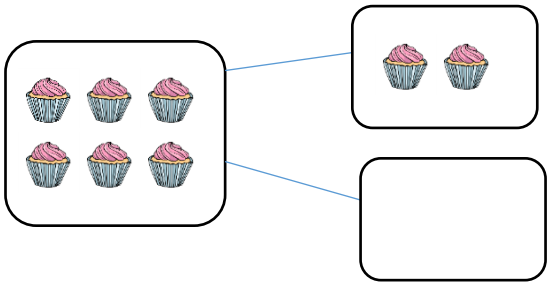
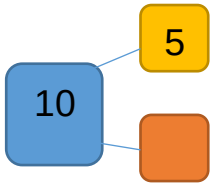
True or false

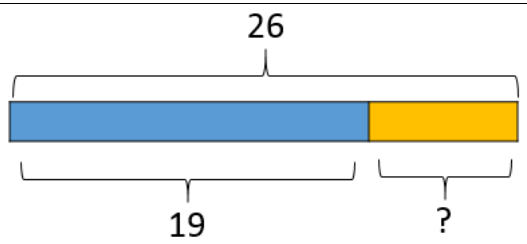
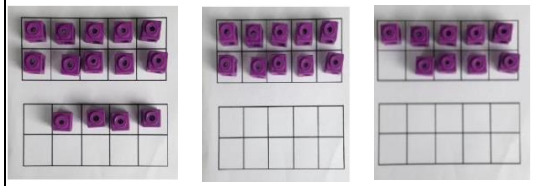
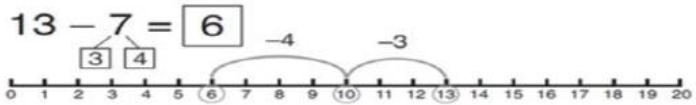
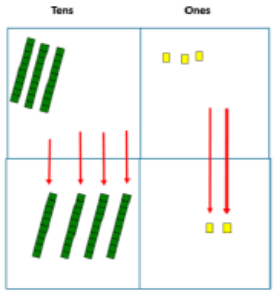
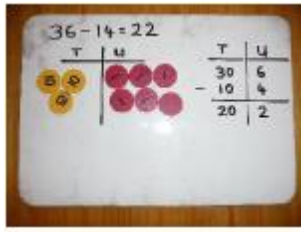
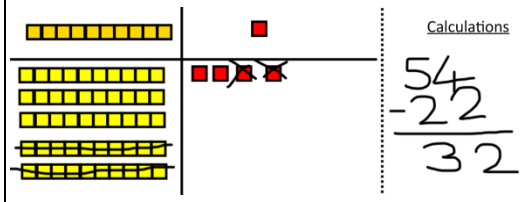
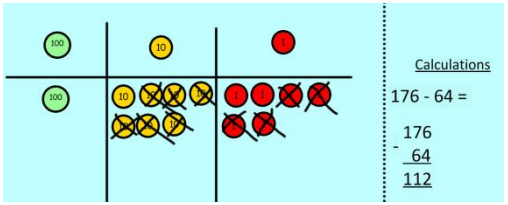
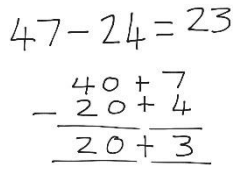
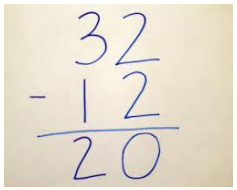
673=527+146

☐ = 527+146

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones takeaway, left, less than, smaller, least, decrease, fewer	Use physical objects, counters, cubes, numicon, abacus, etc to show how objects can be taken away.  6 – 2 = 4   	Children to draw the concrete resources that they are using and cross out drawn objects to show what has been taken away.  15 – 3 = 12 Children can use bar models to show their calculations: 10-4 =  10 X X X X	18 -3= 15 8 – 2 = 6
Counting back Count back, number line, takeaway, smaller	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go.	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. Children could use an empty number line.

	 Use an abacus and make the larger number first and takeaway the ones.	This can progress all the way to counting back using two 2 digit numbers.	
Find the difference difference, between, count on	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers.  	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. Find the difference between 15 and 8.
Part Part Whole Model difference,subtraction , part,whole	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

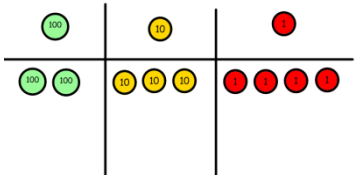
			
Make 10 bonds,partition,take away	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. Children can draw the ten frame pictorially and explain what they did to make the 10 first.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping subtraction, minus, take away, difference	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	 This will lead to a clear written column subtraction. 

Column method with regrouping

subtraction, minus, take away, difference, exchange

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

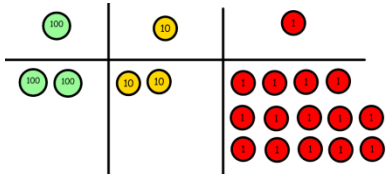
Make the larger number with the place value counters



Calculations

234
- 88

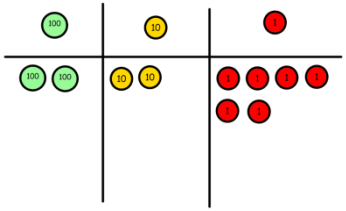
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

234
- 88

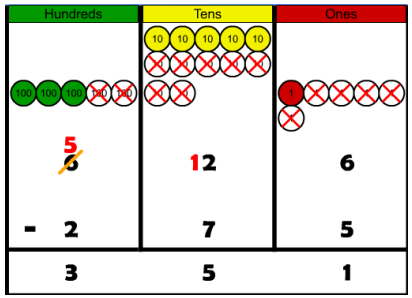
Now I can subtract my ones.



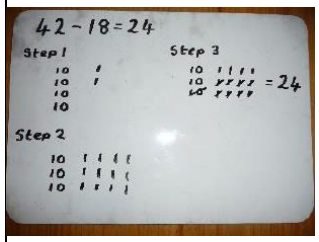
Calculations

234
- 88

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

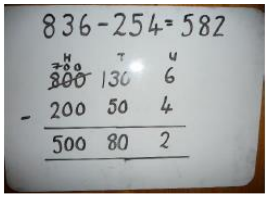


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

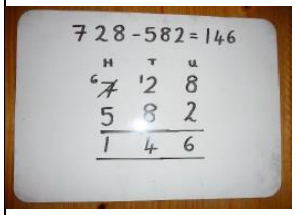


When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.



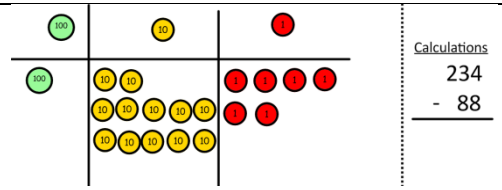
Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.

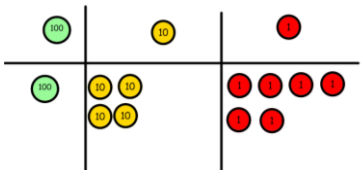
```
      5  12     1
    2  6  3  .  0
-      2  6  .  5
   ---------------
    2  3  6  .  5
```



Calculations

234
- 88

Now I can take away eight tens and complete my subtraction



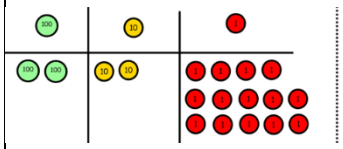
Calculations

~~234~~
~~224~~
- 88
146

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Conceptual variation

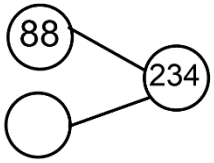
Use of bar models, part whole models, place value charts, word problems and inverse calculations.



Calculations

234
- 88

234	
88	?



Two hundred and thirty four – eighty eight =

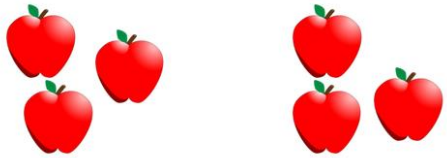
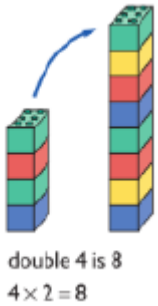
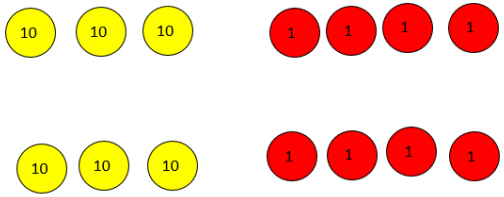
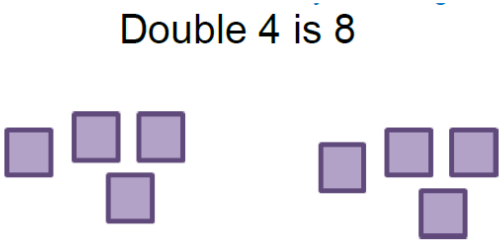
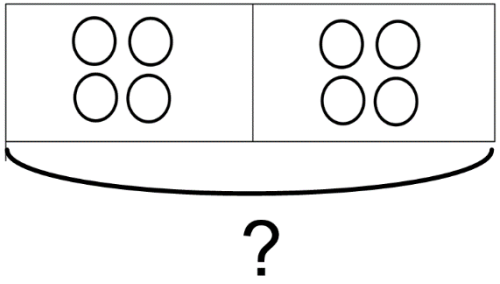
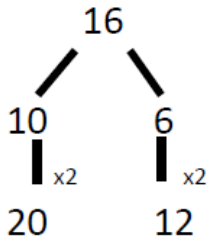
A zoo has 234 animals. The following year 88 move to another zoo. How many animals are left?

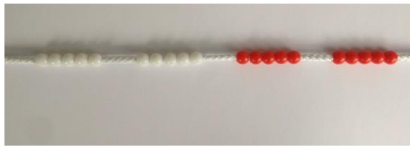
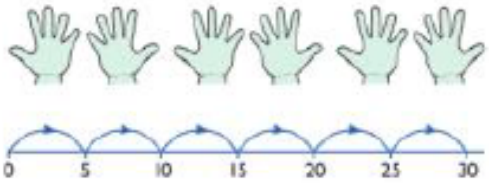
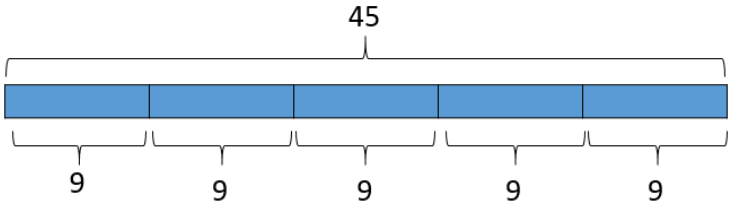
True or false

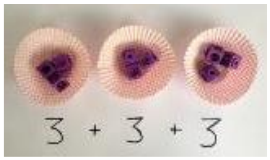
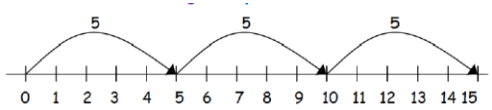
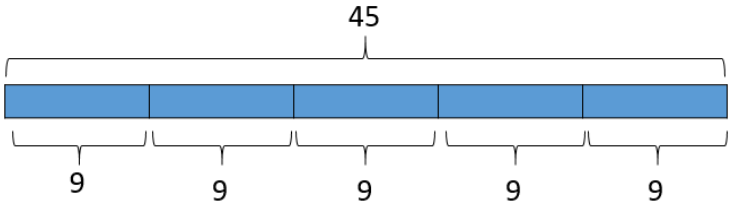
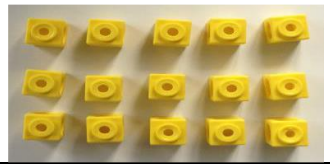
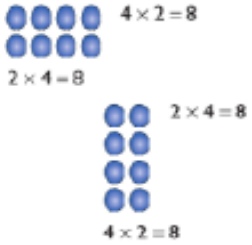
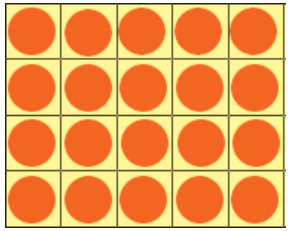
234-88=146

234-□= 88

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling groups, equal groups, How many times?	**Doubling single digit numbers:** Use practical activities to show how to double a number.   **Doubling two and three digit numbers:** 	Draw pictures to show how to double a number.  Children to represent the practical resources in a picture and use a bar model. 	 Partition a number and then double each part before recombining it back together.

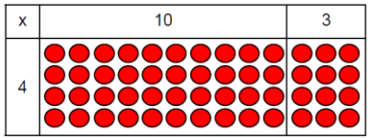
Counting in multiples sequences, multiples, equal groups	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition equal groups, part, repeated addition, grouping, How many times?	   3 + 3 + 3 Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15 	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10
Arrays- showing commutative multiplication multiply, array, times,	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Grid Method

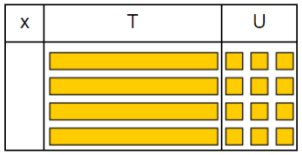
multiply, times, arrays

Show the link with arrays to first introduce the grid method.



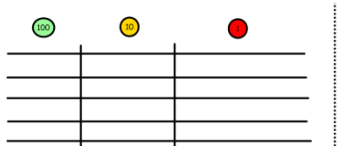
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.



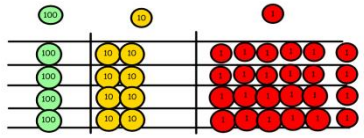
4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



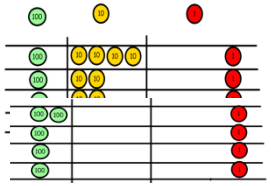
Calculations
4 x 126

Fill each row with 126.



Calculations
4 x 126

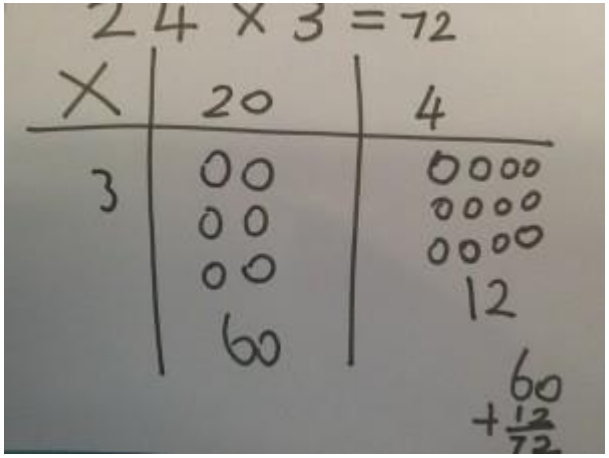
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

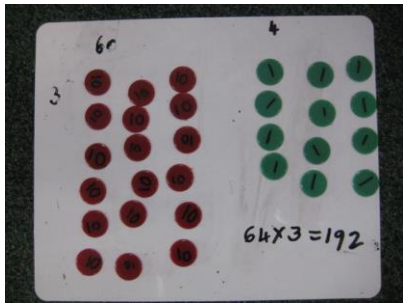
	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Column multiplication

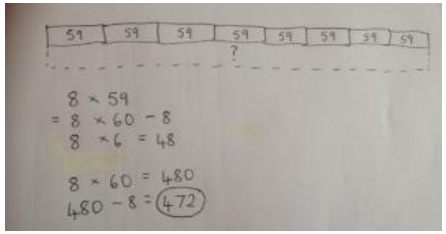
multiply, place value, exchange,

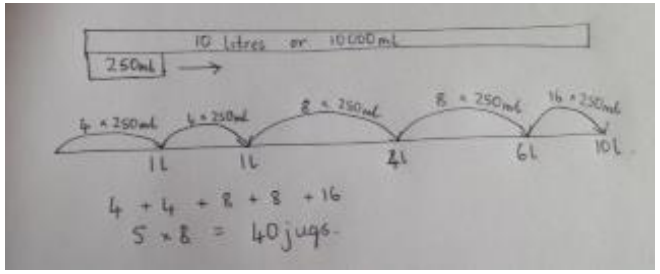
Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.





Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

```
  32
x 24
   8   (4 x 2)
 120   (4 x 30)
  40   (20 x 2)
 600   (20 x 30)
 768
```

```
    74
  x 63
  ----
    12
   210
   240
+ 4200
  ----
  4662
```

This moves to the more compact method.

```
  1342
x   24
  ----
  5368
 1 1
 26840
 ------
 32208
 1 1 1
```

			Or if children struggle to lay this out as one long multiplication calculation, they could do two short-style multiplications and add the results. 1342 x 4 5368 1 1 1342 x 20 26840 5368 +26840 32208 1 1 1

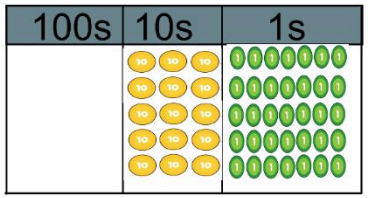
Conceptual variation 37 x 5

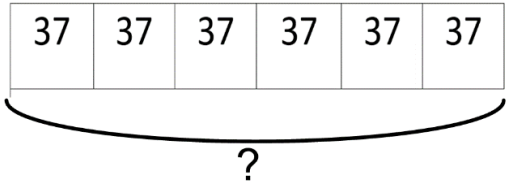
With the counters, prove that 5 x 37 = 185

□ = 5 x 37

Find the product of 5 and 37

What is the calculation?



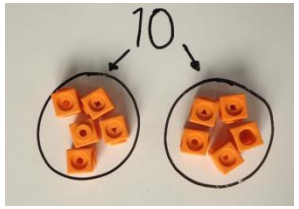
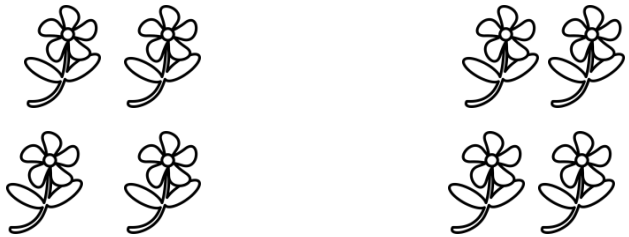


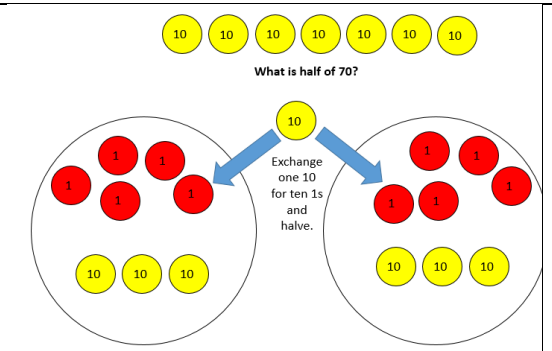
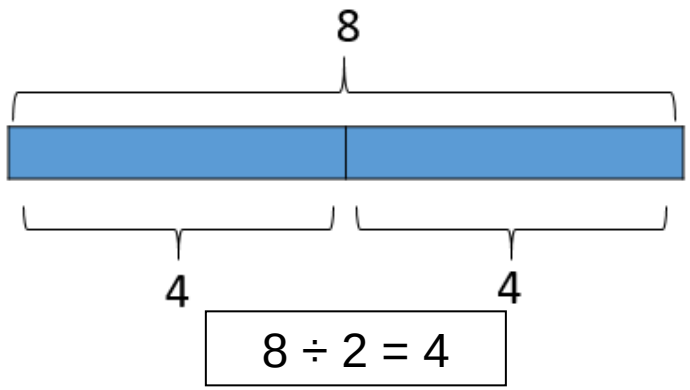
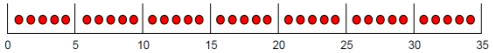
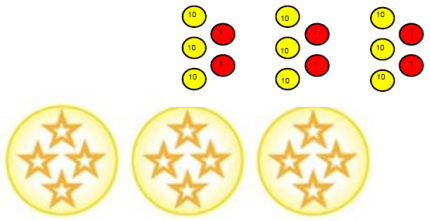
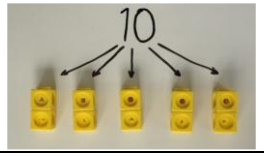
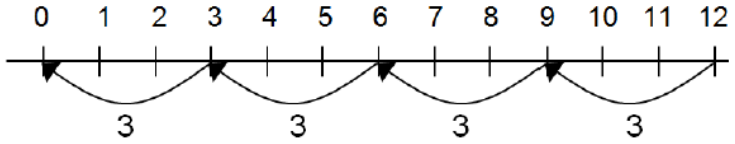
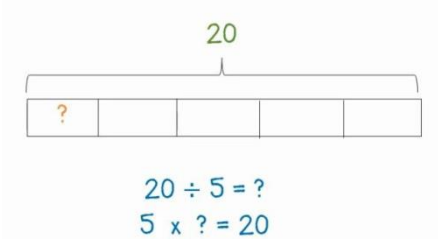
x 5 37 x 37 5

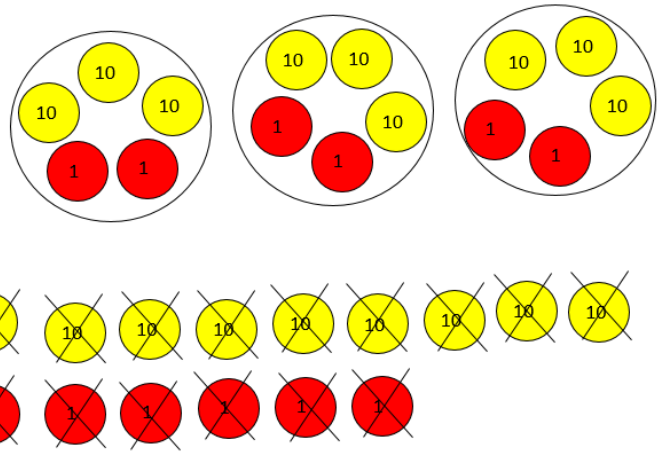
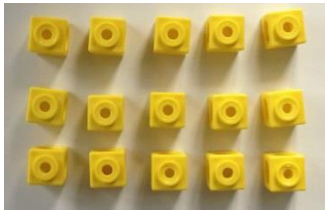
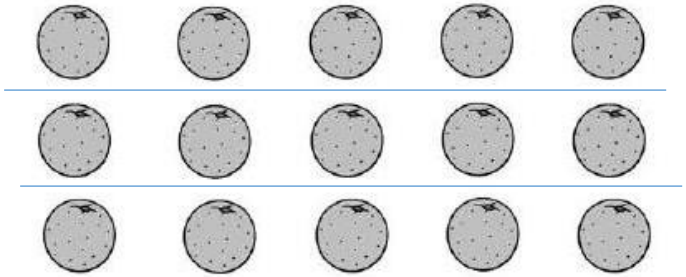
Sarah had to swim 37 lengths, 5 times a week. How many lengths did she swim in one week?

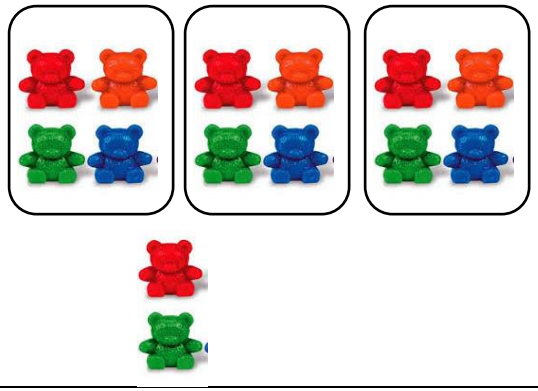
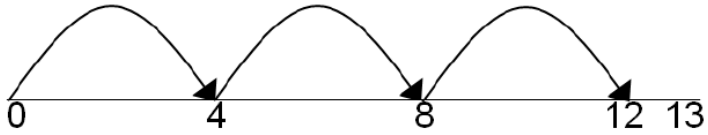
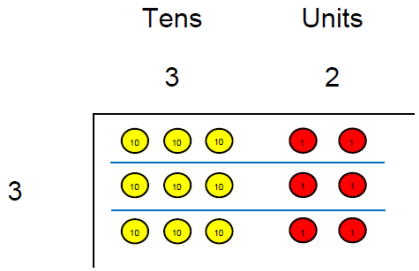
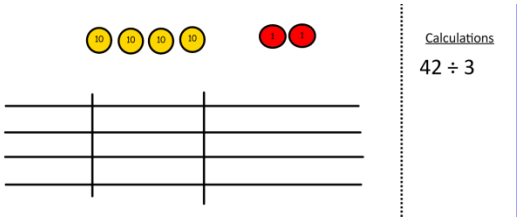
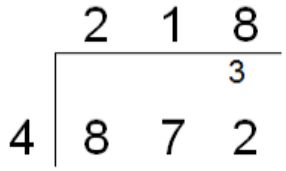
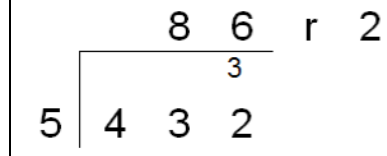
5 x 37 =

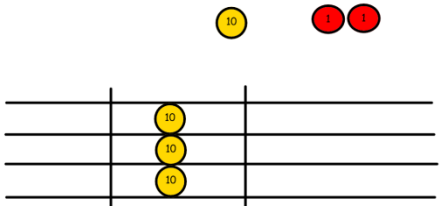
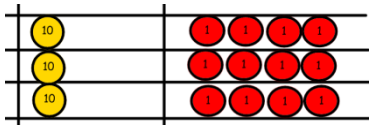
Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups divide, divided by,share,each, equally,group, groups of,lots of,	 I have 10 cubes, can you share them equally in 2 groups?  **Halving odd multiples of 10** (the same principle can be used for odd numbers – exchange a 1 for ten 0.1s)	Children use pictures or shapes to share quantities. 	Share 9 buns between three people. 9 ÷ 3 = 3 Half of 70 = 35

Halving share,equal, groups			70÷2=35
Division as grouping equal groups, grouping,sharing,	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  96 ÷ 3 = 32  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

		Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	
Division within arrays divide, share, groups,	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Division with a remainder divisible by, carry, groups of ___, remainder,	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 
Short division divisible by, carry, groups of ___, remainder,	96 ÷ 3 = 32  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.

	can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.		1 4 r 21 35 $\overline{)}$ $^{4}\not{5}$ $^{1}1$ 1 3 5 ↓ 1 6 1 Children can explore a variety of methods to find multiples of the divisor (in this case 35)

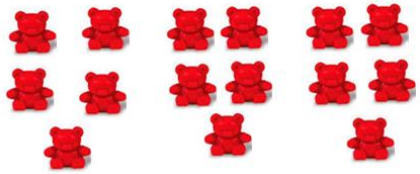
Conceptual variation 15÷3= Use of bar models, part whole models, place value charts, word problems and inverse calculations.

The teacher shares 15 sweets with 3 pupils.

How many sweets does each pupil recieve?

15		
?	?	?

Write the calculation to match the image



With the counters, prove that 15÷3=5

□=15÷3

Gorse Hall Maths Team

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